Problem #498
Solution

Determine whether or not there exist integers \( a, b, c \) and \( d \), such that the last four digits of \((a + b)(b + c)(c + d)(d + a)\) are 2014.

Answer: No such integers exist.

Proof. Observe that 2014 is divisible by 2 and not by 4. If the integers \( a, b, c \) and \( d \) exist, then exactly one of \( a + b, b + c, c + d \) or \( d + a \) is even and the other three must be odd, so the sum of all four factors must be odd. On the other hand \((a + b) + (b + c) + (c + d) + (d + a) = 2(a + b + c + d)\), which is a contradiction. Therefore the integers in question do not exist. \(\square\)

Source: Jacek Fabrykowski.