Problem #503
Solution

In the multiplication problem below, each letter represents a different digit:

\[
\begin{array}{cccccc}
A & B & C & D & E \\
\times & & & & F \\
\hline
G & G & G & G & G & G
\end{array}
\]

Find all possible choices for digits A through G.

Answer: The unique choice is \( ABCDE = 95238 \), \( F = 7 \), \( G = 6 \).

Proof. Note that \( GGGGGG = G \cdot 111111 = G \cdot 3 \cdot 7 \cdot 11 \cdot 13 \cdot 37 \). It follows that \( ABCDE \) is divisible by \( 11 \cdot 13 \cdot 37 = 5291 \), so \( ABCDE \) is a multiple of 5291. If \( F | G \), then

\[
99999 > ABCDE = \frac{GGGGGG}{F} = 111111 \cdot \frac{G}{F} > 1111111,
\]

which is a contradiction. Consequently, one of the two numbers 3 or 7 must divide \( ABCDE \) and the other must divide \( F \).

It follows that

\[
ABCD = k \cdot 5291,
\]

where either 3 or 7 divides \( k \). If \( 7 | k \), then \( ABCDE \) is a multiple of \( 7 \cdot 5291 = 37037 \). However, the only five digit multiples of 37037 are 37037 and 74074, and neither one meets the necessary condition that the digits are all distinct. Thus \( k \) is a multiple of 3, \( ABDCE \) is a multiple of \( 3 \cdot 5291 = 15783 \), and \( F = 7 \). The five-digit multiples of 15783 are

\[
15783, 31746, 47619, 63492, 79365, \text{ and } 95238.
\]

All but 63492 and 95238 can be eliminated because the others all contain the digit 7. Now 63942 \( \star 7 = 444444 \), and this would imply that 4 is represented by both \( D \) and \( G \). The only remaining choice is \( ABCDE = 95238 \), and this gives \( G = 7 \cdot 95238/111111 = 6 \).

Source: New Puzzles in Logical Deduction by George Summers.