Problem #507
Solution

Three numbers $x, y, z$ satisfy the equation

$$x + y + z - 2(xy + yz + zx) + 4xyz = \frac{1}{2},$$

Prove that at least one of $x, y, z$ is equal to $\frac{1}{2}$.

Proof. Observe that

$$4 \left( x - \frac{1}{2} \right) \left( y - \frac{1}{2} \right) \left( z - \frac{1}{2} \right) = x + y + z - 2(xy + yz + zx) + 4xyz - \frac{1}{2} = 0.$$

Therefore, at least one of $x - \frac{1}{2}, y - \frac{1}{2}, z - \frac{1}{2}$ must be 0. □

Source: Moscow Mathematical Olympiad 1998