Let $p$ be a prime, $p \neq 3$, and let $a$ be a positive integer. Suppose $p^a$ has 20 digits when written in base ten. Prove that at least one digit appears at least three times in this base ten representation.

Rules

1. Anyone is eligible to participate. Each solution is to be the work of one individual without any input from faculty or others. An answer must be accompanied by appropriate justification to be considered correct.

2. The solution is to be submitted with the solver’s name, email, year in school (if applicable), local phone number, and local address. If you are submitting this for possible credit in a class, include your class number and instructors name.

3. The solution is to be typed or legibly written. Solutions must be submitted to the Mathematics Department Office (PE 214) by 2 p.m. on the due date.

4. Entries will be graded on clarity of exposition and elegance of solution. An award of $10 will be given for the best correct solution. In the case of a two-way tie, the award will be split. If there are more than two best solutions, a drawing will be held to determine two award winners. If no correct solution is presented by an undergraduate student, the award will be carried over to the following week.

5. Graduate students, faculty, and members of the general public are encouraged to submit solutions, but they will not be considered for the monetary award.

6. Names of all solvers will be posted across the Mathematics Department Office within one week of the due date.