Aug 25, 2017 Applied Statistics Qualifier: STA 682 **Name _____________________________**

There are 3 problems. You may submit your answers on the additional paper provided.

1. Consider a simple linear regression model $Y = \beta_0 + \beta_1 X + \varepsilon$ in which ε are independent identically distributed random variables $N(0, \sigma^2)$. Let $Y^* = cY$ where c is a non-zero constant.

   a) Show the effect that Y^* has on the estimates on β_0 and β_1.

   b) When testing H_o: $\beta_1 = 0$, compare $t^* = \frac{b_1^* - \beta_1}{SE_{b_1^*}}$ to $t = \frac{b_1 - \beta_1}{SE_{b_1}}$. Prove your results.

   c) Compare R^2 using $Y^* = cY$ with R^2 using just Y. Prove your result.

2. Consider the following linear models, in matrix notation, of [Y] = [XB] versus [Y] = [XB Zδ].

   a) If [Y] = [XB Zδ] is the true model, show/discuss the results of fitting [Y] = [XB] on the parameter estimates (bias) , estimates (Yhat), S, and diagnostics. This is of course an underparameterized model.

   b) If [Y] = [XB] is the true model, show/discuss the results of fitting [Y] = [XB Zδ] on the parameter estimates, model performance, and diagnostics. This is of course an overparameterized model.

   c) In your opinion, which of the two misspecifications (under parameterized or over parameterized) is a more difficult problem to detect.

3. 3 pieces of gold, weighing a total of 100 grams are weighed 3 times on a non-accurate scale. Here are the results:

Scale	Piece A	Piece B	Piece C
1	30	20	50
2	35	25	55
3	25	20	45

In addition, to assist in estimating the weights, a very accurate double pan balance scale, with two pans like the ones shown below, is being used to compare the 3 separate pieces of gold.

Here are the results of the measurement experiment:

Piece	Piece	Difference
A	B	A: +5 gram
A	C	A: +2 gram
B	C	B: +1 gram

Consider the results of the double pan scale a constriction on the model. Also, as previously mentioned, assume that the 3 pieces weigh a total 100 grams. Given this information, use Least Squares in matrix format to do the following.

a) State the Response Matrix, Design Matrix, Parameter Matrix, and any other matrices needed to create this estimation problem as a constricted linear model.

b) Estimate the weight of each of the 3 pieces using this model, subject to the constrictions as previously specified.