# Solution to problem # 656

**PROBLEM:** Find all real $x$ and $y$ such that

$$\cos^2 x = \frac{(1+y)^2}{4y}.$$

**SOLUTION**: Since $\cos^2 x \geq 0$, we must have $y > 0$. Now

$$\frac{(1+y)^2}{4y} = \frac{1+2y+y^2}{y} = \frac{1}{4}\left(\frac{1}{y} - 2 + y\right) + 1 = \frac{1}{4}\left(\frac{1}{\sqrt{y}} - \sqrt{y}\right)^2 + 1,$$

so the right hand side of the given equation is $\geq 1$ and equal to 1 if and only if $\dfrac{1}{\sqrt{y}} = \sqrt{y}$, i.e. $y = 1$. On the other habd $\cos^2 x \leq 1$, so the only way the two sides can be equal is if both are equal to 1. Therefore $\cos^2 x = 1$, i.e. $\cos x = \pm 1$. It follows that the possible values of $x, y$ satisfying the above equation are

$$y = 1, \quad \text{and } x = n\pi \text{ for an integer } n.$$