# Solution to problem # 658

**PROBLEM:** It is known that two of the roots of the cubic equation $x^3 + 3x^2 - 9x + c = 0$ are equal. If $c > 0$, find the value of $c$.

**SOLUTION**: Let the roots be $\alpha, \alpha$ and $\beta$, then we have

$$\begin{aligned} x^3 + 3x^2 - 9x + c &= (x - \alpha)^2(x - \beta) = (x^2 - 2\alpha x + \alpha^2)(x - \beta) \\ &= x^3 - (\beta + 2\alpha)x^2 + (2\alpha\beta + \alpha^2)x - \alpha^2\beta \end{aligned}$$

Equating the coefficients we obtain the equations $\beta + 2\alpha = -3, \quad 2\alpha\beta + \alpha^2 = -9, \quad \alpha^2\beta = -c$. From the first equation $\beta = -3 - 2\alpha$, which when substituted in the second equation gives the quadratic $\alpha^2 + 2\alpha - 3 = 0$, so that $\alpha = -3$ or $\alpha = 1$. If $\alpha = -3$, then $\beta = 3 - 2\alpha = 3$, so $c = -\alpha^2\beta = -27$, so this is not possible. If $\alpha = 1$, then $\beta = -3 - 2\alpha = -5$, and so $c = -\alpha^2\beta = 5 > 0$, so we must have have $c = 5$.