Mathematics Education Qualifying Exam
August 2014

The following questions constitute the Mathematics Education Qualifying Exam for August of 2014. The questions are separated into two sections; the first section is based on MTH 761 and the second section is based on MTH 762. You must answer both questions in Section I and two of the three questions in Section II. In Section II, make sure it is clear which questions you are answering and would like considered. You have four hours to complete this exam. Remember to save your work frequently. The vignette for question 2 of Section I is located at the end of this document for your convenience as well as in print form.

Section I:

1. Sfard's (1991) model for mathematical concept development contains three stages: interiorization, condensation, and reification.
   a. Describe each of these stages and discuss how the student progresses through them giving examples to illustrate your points.

   b. Kaput. Blanton, and Moreno (2008) describe a model for the development of symbolic meaning. Compare and contrast their model with that of Sfard's (1991) paying particular attention to semiotics (symbol systems).

2. Consider the case study, *Inverse Functions: The Case of Tim*. In this classroom exchange, Tim is teaching the concept of inverse functions.
    a. Leinhardt, Zaslavsky, and Stein (1990) discuss the role multiple representations play in the development of the concept of function. In particular, they discuss the difficulty students have in translating between graphs and equations with the translation from graph to equation as being the more difficult than equation to graph. With respect to their positions on translation among representations, explain how the teacher, Tim, might have approached the lesson differently so that a deeper connection might have been made by the students. Be certain to cite relevant research to support your positions. Do not limit yourself to only Leinhardt, Zaslavsky, and Stein, but also include your thoughts based on other research you have read.

    b. Using what you know from the research literature about the development of mathematical understanding and the use and evolution of symbol systems, describe what you see as the main issues stemming from Kat's questions and Tim's responses to them as they relate to Sfard's (1991) position that abstract mathematical notions can be conceived in two fundamentally different ways: structurally and operationally. Do not limit yourself to only Sfard's perspective, but make sure to include a treatment of it in your response.

Section II:

1. The study by Heid (1988) uses what is known as "mixed methods". This means that the study has both quantitative and qualitative components. Discuss the different methodology used for each component of the study. Then discuss in detail some of the things that this scholar learned from the quantitative portion of the study that could not have been discovered by the qualitative portion alone and some things that were discovered by the qualitative portion of the study that could not have been discovered by the quantitative portion alone.

2. Heid's (1988) paper is based on results from her dissertation study. When designing a research study in mathematics education, there are many facets a researcher must consider prior to conducting the study including: clear research questions, well-chosen variables, carefully selected research methods, a plan that must address reliability and validity in the data collection and analysis, and a thoughtful procedure for analyzing the data. If you were to conduct a modified form of this study for your dissertation, what are some of the aforementioned facets that you would address differently than Heid? Make sure to provide solid reasons for your responses.

3. The scholarly literature on mathematical proof suggests many reasons why college students have difficulties with the transition from computational mathematics to proof writing. Discuss some of the causes for this difficulty as found in the literature.

Inverse Functions: The Case of Tim

Tim is a beginning teacher in his second year. Today, he is teaching the concept of inverse function. To find an inverse function, the text that Tim is using begins by having the students swap the variables of *x* and *y* and then solve for *y*. He is fairly confident that he can teach his students how to use this procedure to find inverse functions, but also noted that the book has students check to see if the inverse function works by composing the two functions to see if the composition results in "*x*". Tim also noticed that the book covers function composition in the section before inverse function and assumes this is so that students can understand how to check their answers once they have found the inverse function. Since he wants to be sure the students are ready, he begins class by discussing the previous day's lesson on function composition.

Tim: OK, let's make sure we understand what we did yesterday since we are going to use it today. What is composition of functions?

Jen: Isn't it just sticking one thing into another?

Tim: Yes, it is. Can anyone show us how to do it? [Lori raises her hand]. Lori, go to the board. Can anyone give us two functions? [Ken raises his hand]. Ken, shoot.

Ken: How about $f(x) = 3x - 1$ and $g(x) = x^2 + x$?

Tim: Great. Now Lori, show us what to do.

Lori: OK, I guess it depends on what order you want.

Tim: Let's do $f(g(x))$.

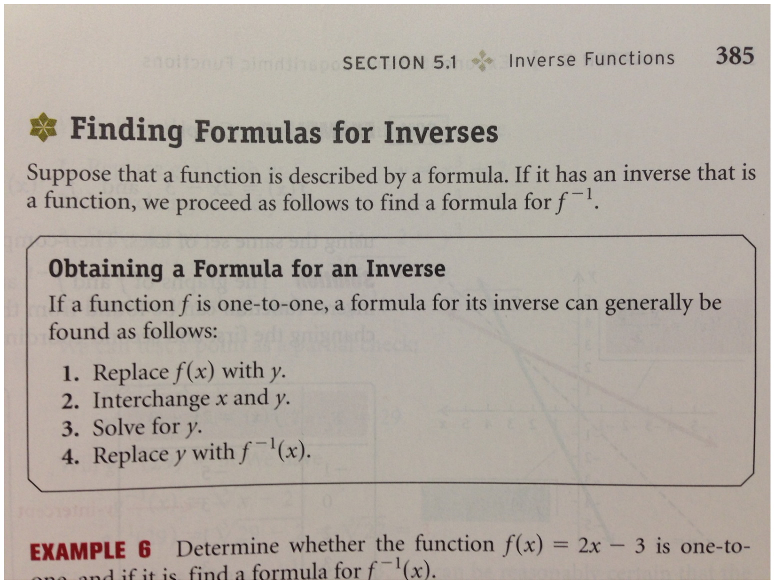
Lori: So we just take $x^2 + x$ and stick it in for *x* in $3x - 1$. That gives us $3(x^2 + x) - 1$. Now we can just distribute it out and that gives us $3x^2 + 3x - 1$.

Tim: Super! Is everyone OK with that? [The class is quiet]. Good, let's move on to today's stuff. Thanks, Lori. [Lori sits down].

Tim: Today we're going to look at inverses when it comes to functions. Has anyone ever heard the word "inverse" before? [Several hands go up]. Yeah, Mia.

Mia: Wouldn't that just be like the flip of a function?

Tim: [Smiles]. No, that's a common mistake. This is completely different when it comes to functions. Let's look at the book's way of finding inverse functions. In pairs, take a couple of minutes to read the procedure on page 385.



SECTION 5.1 Inverse Functions 385

Finding Formulas for Inverses

Suppose that a function is described by a formula. If it has an inverse that is a function, we proceed as follows to find a formula for f^{-1}.

Obtaining a Formula for an Inverse

If a function f is one-to-one, a formula for its inverse can generally be found as follows:

1. Replace $f(x)$ with y.
2. Interchange x and y.
3. Solve for y.
4. Replace y with $f^{-1}(x)$.

EXAMPLE 6 Determine whether the function $f(x) = 2x - 3$ is one-to-one, and if it is, find a formula for $f^{-1}(x)$.

Section from Tim's Text

[Students are given 5 minutes to read and discuss the procedure as Tim circulates observing their discussions. Tim notices that in the pair of Mike and Sam, Mike seems to be doing most of the explaining and so plans to call on Mike.]

Tim: Mike, in your own words explain how to find inverse functions.

Mike: Well, first you just switch the *x* and *y*s. Then you just solve for *y* so that *y* is on one side of the equation and the other side just has *x*s.

Tim: Very good. Let's do one together. Let's start with lines. How about the function we used earlier, $y = 3x - 1$. Tell me what to do. Yeah, Joan, get us started.

Joan: First write it as $x = 3y - 1$.

Tim: Now what? Someone else [Hiro raises his hand]. Hiro go ahead.

Hiro: Just add 1 to both sides to get $x + 1 = 3y$. The only thing we have to do now is divide by 3 so we get *y* all by itself. That gives us $y = \frac{x+1}{3}$.

Tim: Excellent. Does everyone see how it's done? [Kat raises her hand]. Yeah, Kat.

Kat: How do we know it's the inverse function?

Tim: I'm glad you asked that, Kat. That's our next part of the lesson. To see if it worked, we just think of these as functions and compose them like we did yesterday. If we get "*x*", then they are inverses. Let's try it. OK, let's make $f(x) = 3x - 1$ like we did before and now make $g(x) = \frac{x+1}{3}$. Sue, come to the board and work out this composition.

Sue: [Sue goes to the board and begins writing.] OK, we have $f(g(x))$. That gives $3\left(\frac{x+1}{3}\right) - 1$. We can cancel the 3s and that leaves just $x + 1$ and -1. The 1 and -1 cancel and we get *x*.

Tim: Great. Do you see, Kat? Since we got *x* that means they're inverses.

Kat: I'm still not sure how you know they are inverses?

Tim: Well, we got *x*. If you compose a function and its inverse you get *x*. Let's do a few more and that might help with remembering it.

Kat: But why do we get *x*? Why don't we get 1?

Tim: That's just the definition of what it means to be an inverse function. Just follow these steps and you'll be OK.

[Kat gives a frustrated look and pulls out another sheet of paper.]