Mathematics Education Qualifying Exam
August 2016

The following questions constitute the Mathematics Education Qualifying Exam for August of 2016. The questions are separated into two sections; the first section is based on MTH 761 and the second section is based on MTH 762. You must answer both questions in Section I and two of the three questions in Section II. In Section II, make sure it is clear which questions you are answering and would like considered. You have four hours to complete this exam. Remember to save your work frequently. The Calculus sequence of topics for question 1 and vignette for question 2 of Section I are located at the end of this document for your convenience as well as in print form.

Section I:

1. When looking at the text by James Stewart, *Essential Calculus: Early Transcendentals (2nd Ed.)*, an instructor decided to alter the sequence of the sections as she prepared for a 15-week Calculus 1 course. She is using a computer algebra system in teaching this course and chose to teach the sections in the order given at the end of this document (note the text's original section numbers are given so that you can see the order in which topics were arranged by the author). Discuss your thoughts on her decision by providing research-based rational for your opinion.

2. Consider the case study, *So Many Ways—So Little Time*. In this classroom exchange, Joan is teaching the concept of solving linear equations in one variable. Sfard's (1991) model for mathematical concept development contains three stages: interiorization, condensation, and reification.
    a. Describe each of these stages and discuss how Joan has designed her lesson to aid the students' progress through them giving examples to illustrate your points.

    b. Kaput. Blanton, and Moreno (2008) describe a model for the development of symbolic meaning. Compare and contrast their model with that of Sfard's (1991) paying particular attention to semiotics (symbol systems). Illustrate your thoughts by referring to situations from the vignette of Joan's algebra class.

    c. At one point a student, Sue, offers a path to the solution of the equation that begins with dividing both sides of the equation by 5. Discuss your thoughts on how the teacher handled the suggestion supporting your opinion (whether positive or negative) with what you know from the research literature.

Section II: Choose two of the three items below for this portion of the exam.

1. Rina Zazkis and Peter Liljedahl (2004) wrote what kind of paper? That is, was it known as "mixed methods", quantitative or qualitative study? Explain your selection, then discuss the different methodology and instruments used for this study. Then discuss in detail some of the things that this scholar learned from the study. Make sure your discussion goes beyond what is found in the abstract. Finally, suggest improvements and provide reasons for these improvements.

2. What are some of the typical (often used) instruments and data collection methods for each of these kinds of studies. Provide at least four for each type of study and give a short explanation for the possible value of each instrument for the research. The study types are quantitative, qualitative and mixed-methods.

3. Recall Schoenfeld's list of "Standards for Judging Theories, Models and Results. Sfard and Dubinsky's theories were mentioned. Use the seven in his list. Also, where if any was there evidence or statements in this paper that addressed Schoenfeld's question, "How much faith should one have in a particular result?".

Instructor's Reordering of Calculus Topics

§2.1: Derivatives and Rates of Change
§2.2: The Derivative as a Function
§4.1: Maximum and Minimum Values
§4.2: Mean Value Theorem
§4.3: Derivatives and the Shapes of Graphs
§4.5: Optimization Problems
§4.6: Newton's Method
§1.3: The Limit of a Function
§1.4: Calculating Limits
§1.5: Continuity
§1.6: Limits Involving Infinity
§2.3: Basic Derivative Formulas
§2.4: Product and Quotient Rules
§2.5: Chain Rule
§2.6: Implicit Differentiation
§2.7: Related Rates
§2.8: Linear Approximation and Differentials
§3.1: Exponential Functions
§3.2: Inverse Functions and Logarithms
§3.3: Derivatives of Logarithmic and Exponential Functions
§3.4: Exponential Growth and Decay
§3.5: Inverse Trigonometric Functions
§3.6: Hyperbolic Functions
§3.7: Indeterminate Forms and L'Hopital's Rule
§4.7: Antiderivatives
§5.1: Areas and Distances
§5.2: The Definite Integral
§5.3: Evaluating Definite Integrals
§5.4: Fundamental Theorem of Calculus

Case Study: So Many Ways—So Little Time.

Joan's Algebra I class has been working with pan balances to explore solving linear equations in one variable. Joan's goal for today's class is to take the students' physical processes in the solution of equations and help them convert them to algebraic processes. She sees her task as focusing on two main ideas: undoing and order of operations. For homework last night, students were given the assignment of solving some equations using blocks and their physical processes developed in class. Since the students do not each have a pan balance at home, the teacher gave each student two paper plates to use for the two sides of a balance and instructed them to use their imagination, knowing from class experience what processes would leave the two plates "in balance". We join the discussion at the beginning of class on the next day.

Joan: So, how did the homework go last night? Were you able to solve the equations I gave you?

Kali: I think I got'em all.

Joan: You think? Is there a way you could be sure?

Kali: I don't know. How could I be sure?

Joan: Does anyone have an idea for how Kali could be sure her answers are correct?

Derek raises his hand about halfway up. During yesterday's class, Joan observed Derek writing down the answers he was finding with the balance and calculating both sides of the equation to check his work. She is fairly confident Derek will share his method with the class even though Derek seems unsure himself.

Joan: Derek?

Derek: Well, I think you can plug in the answer and see if it works. At least that's what I did yesterday 'cause I didn't trust the balance.

Joan: What do you mean, 'didn't trust the balance'?

Derek: Well, sometimes you couldn't tell if the balance was really balanced or not. I mean, it could be balanced, but just off a bit 'cause the needle didn't line up exactly with the mark. Or maybe the blocks we used weren't all the exact same weight. You know—error and stuff.

Joan: That raises an interesting question though, Derek. If the balance or blocks can be off a bit, do we need to come up with a better way to solve equations that doesn't rely on physical things? We'll get back to that question in a minute, but for now, tell us how you checked your answers.

Derek: Well, I just took the answer and plugged it into both sides of the equation. If both sides matched, then I knew it was right.

Joan: What do you think of Derek's idea? Does it address his problem with the balance or blocks being a bit off? [*Shelby raises her hand.*] Shelby?

Shelby: I think it would fix it. When we go back to the equation we started with, we haven't used any blocks or balance so it shouldn't matter if there's error in the blocks.

At this point, Joan is happy the students are agreeing with checking both sides of the equation. Her agenda for tomorrow's class is to look at the graphs of both sides of the equation and explore the intersection points. She can now use Derek's suggestion to connect the common y-coordinates of the graphs to the output of the left and right hand sides of the equation as Derek has suggested.

Joan: That's a good observation, Shelby. So does this allow us to not worry about the errors involved with balances, blocks, plates, etc.? Yeah, Lashawn?

Lashawn: Not really.

Joan: Explain what you mean.

Lashawn: I mean, just 'cause the answer checks out not using blocks and stuff, how do you know if the way you did it will always give right answers. I mean, if we knew the answer to start with, we wouldn't have to do it in the first place. Checking answers won't tell you if the *way* we found it using the balance and stuff can be done without'em.

Joan: Good point. Does everyone see what Lashawn is saying? [*Class nods their heads seeming to agree with Lashawn*] Is there a way we can turn what you did last night with the plates into a process we can do with algebra? Since you actually didn't have a balance last night for homework and just used your imagination with the plates, couldn't we just use our imagination with the equation to do the same thing? [*Pause*]

Joan: Can anyone give me a "nutshell" explanation of what you did with the plates? What were the basic moves you used to solve the equations? Yeah, Jen.

Jen: It really was just undoing stuff and making sure you did the same things to both sides of the equation to keep it balanced.

Joan: That's a nice way to put it. Maybe we could start with an equation and try what Jen just suggested? How about $5x - 3 = 2x + 6$? [*Joan writes the equation on the board*] What should I do?

Sue: How about dividing both sides by 5? That'll undo the 5 in the $5x$.

Joan: Well, that's a nice try, but it won't work. Can anyone tell us why not based on what we did yesterday with the balance?

Ana: We need to do the right order of operations. How are you gonna divide the 6 blocks on the right side by 5? Since the last thing done on the left hand side is subtracting, we need to undo it first in order to get to the answer. Then we can undo the multiplication.

Roberto: Let's undo the minus 3 by adding 3 to both sides. That leaves us $5x = 2x + 9$. Now we can take away the $2x$ off both sides of the balance, that way we get $3x = 9$ and we can divide up the 9 into 3 piles to get 3 in each pile. The answer is 3.

Joan: Very nice, Roberto. Does everyone follow what Roberto and Ana are saying? [*Most students nod*] Should we do some more to practice?

Sue: I'm still not sure why we can't divide by 5 first?

Joan: Let's just do some more and you'll see why as you do them.

[*Sue seems frustrated and pulls out some paper to start practicing.*]